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POLYTECHNIC INSTITUTE OF BROOKLYN

Final Report

ACTIVE AND TIME VARYING SYSTEMS AND LINEAR NETWORKS

Report No. PIBMRI-1088-62

Contract No. AF-18(603)-105

Principal Investigator: John G. Truxal

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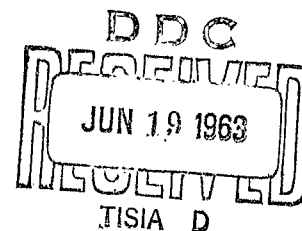
under Contract/Grant

AF 18(603)-105  
PIB (Truxal)

Prepared for

Office of Scientific Research

Washington, D. C.



The Air Force Office of Scientific Research initiated this program with the Polytechnic on July 1, 1956. This program of research in the general field of "Active and Time Varying Systems and Linear Networks" was superseded on 1 July 1962 by Air Force Office of Scientific Research Grant AF-AFOSR 62-280 which provides for continuation of the research program.

Summary of Progress:

The research effort at the Polytechnic has been grouped into two major efforts, namely, "Active and Time Varying Systems Research" conducted by the Department of Electrical Engineering and "Linear Networks Studies" under the cognizance of the Electrophysics Department of the Polytechnic. This summary is organized under these two major headings.

I. ACTIVE AND TIME-VARYING SYSTEMS

A. Background

Research under this contract has been directed toward the development of feedback theory and its application to the analysis and design of complex control systems:

(1) The study of the sensitivity function as a measure of feedback system performance has proceeded, with the development in a doctoral thesis (Report PIBMRI-944-61) by R. A. Haddad of theorems for the evaluation of parameter margins for stability, with a single variable parameter and with several variable parameters.

(2) Criteria for the short-time stability of time-varying systems have been developed in a doctoral thesis by P. Dorato. These results have been published in a research report (PIBMRI-908-61) under this contract and presented at the I. R. E. International Convention in March 1961.

(3) Decision theory has been applied to the design of control systems in a doctoral thesis and report (PIBMRI-945-61) by S. Horing.

(4) Visiting Professor Jens Jonsson (Chairman of the Electrical Engineering Department at Brigham Young University) studied the design of feedback systems with time-varying parameters, with high-speed computational models of the controlled process employed in the operation of the system.

### B. Discussion

(1) Studies under this contract, and by graduate students working for faculty members under the program, have led to extensions of the interpretations of the sensitivity function for the evaluation of complex feedback systems in which specific parameters vary with time or environmental conditions, and also to theorems for the evaluation of parameter margins (measuring the magnitude of change required in a specific parameter to cause the system to break into oscillation). The work will be extended to cover the utilization of such parameter margins and the parameter phase-margins in the evaluation of pertinent parameters in feedback systems--i.e., in the solution of particular identification problems. Of special interest, are the problems associated with the dependence of identification on the feedback configuration. In other words, the significance of errors in identification depends not only on the relations between time and frequency domains, but also on the location of the element within the feedback configuration. For example, if feedback exists around the element, an error in identification may indicate stability of a system which is actually unstable or undesirably close to instability.

Closely associated with the work relating sensitivity to stability considerations is the continuing investigation of the relation between sensitivity and the design techniques appropriate for the feedback system with varying parameters. The control of sensitivity by proper selection of the feedback configuration has been the subject of a large portion of the earlier investigations under the contract; this work is extended to the consideration of the simultaneous, correlated and uncorrelated variation of two or more parameters, ways of compensating for such time variations by the intentional insertion of variable parameters in the controller, and the influences of such simultaneous variations on the stability of the overall system as well as on the dynamic performance characteristics. The investigation of this multi-parameter problem provides the most significant exploitation of the earlier work under this contract, and by other research and groups in the feedback control field, on the general subject of the sensitivity function as a fundamental measure of the quality of the feedback system.

(2) The stability analysis of adaptive control systems generally involves nonlinear, or at best, linear time-varying systems. In studying the stability of linear time-varying systems, the second method of Lyapunov holds great promise and, indeed, has been exploited in studying stability of linear time-varying systems operating over a finite time interval.<sup>1</sup>

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1. Peter Dorato, "Short-Time Stability in Linear Time-Varying Systems," Research Report No. PIBMRI-908-61, May 9, 1961.

If the time variation is of a periodic nature the stability analysis is greatly simplified. Problems where the time variation is periodic occur in some important applications, e. g., the attitude control of satellites. By a simple linear transformation, periodic systems of differential equations can be reduced to differential equations with constant coefficients. The studies emphasize the stability analysis of linear time-varying systems with periodic coefficients.

Time-varying differential equations also appear when studying first-order approximations of nonlinear systems. Thus, the Lyapunov method for linear time-varying systems can also be applied to this case. Some refinements of the Lyapunov method, as indicated in the tests of Cesari<sup>2</sup> and Hahn<sup>3</sup>, were also studied for possible application to this problem.

(3) The construction of a model was undertaken to demonstrate and to evaluate the following:

- a) New techniques for process identification. One of these techniques involves the determination of the system transfer function from sampled measurements of input and output signals taken over a finite time interval. By appropriately weighing the resulting data, the Fourier transforms of the signals involved can be determined with reasonable accuracy, even though the duration of measurements is relatively small. The accuracy of the resulting measured transfer function is greater than that of either of the transforms used to determine it.
- b) Application of learning techniques to control systems. Various learning techniques can be demonstrated using the foregoing model. These learning techniques include, as basic building blocks, the process identification techniques discussed in (a) as well as new pattern recognition techniques which have been developed at the Institute.
- c) Applicability of new methods for the optimum design of control systems. The application of linear decision theory to the design of optimum control systems has been demonstrated in a dissertation at the Polytechnic entitled "On the Optimum Design of Predictor Control Systems" by S. Horing. The ideas presented in this dissertation will be applied to the model under consideration.

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- 2. L. Cesari, "Asymptotic Behavior and Stability Problems in Ordinary Differential Equations", Springer-Verlag, Berlin, 1959.
  - 3. W. Hahn, "Theorie und Anwendung der Direkten Methode von Lyapunov", Springer-Verlag, Berlin, 1959.

## II. LINEAR NETWORKS

### A. Background

The subject of linear networks was treated with the emphasis on two topics. (1) General network realizability of passive systems (2) The network theory of low frequency nonreciprocal circuits and the single frequency behaviour of general linear circuits.

As a result of the realizability investigation a definitive paper was written on the frequency characterization of networks which satisfy basic physical postulates in the time domain. This has led to the extension of the concept of positive real immittance, and bounded real scattering matrix to distributed parameter, nonreciprocal, systems. The work was published in the paper "Scattering Matrices and the Foundations of Linear Passive Network Theory" by D. Youla, L. Castriota and H.J. Carlin. Other work along these lines has considered extensions of some of these ideas to active systems from the point of view of stability.

The program on low frequency nonreciprocal networks has concentrated on the circuit theory of semi-conductor Hall plates. In this context the optimum operating capabilities (from the point of view of insertion loss, isolation, etc.) of devices, such as one-way lines and circulators constructed by imbedding a Hall plate into a resistive or reactive structure, have been determined. A complete theory and tabulation of computed and experimental results appeared in a doctoral study carried out under this program in "Network Theory and Semi-conductor Hall Plate Circuits" by J. Garg. Further work on nonreciprocal network theory is represented by the Doctoral Dissertation carried out under the supervision of H.J. Carlin, "Cascade Synthesis of Reciprocal and Non reciprocal Lossless Two-Ports" by W. Rubin.

At a single frequency a linear time-invariant  $n$ -port  $N$  has two outstanding global attributes which are discernible without a detailed investigation of its fine structure - its nonreciprocal character and its ability to generate or absorb average power. The first property enables it to transmit in preferred directions and the second enables it to serve either as an active or passive device.

Nonsingular lossless, reciprocal  $2n$ -port embeddings which leave the number of ports unchanged keep these properties intact and merely serve to modify them in various ways. All this suggests that the intrinsic performance capabilities of such an  $n$ -port are very intimately related to its invariants under transformations of this kind (which form a group). That this is indeed the case for  $2$ -ports was demonstrated for the first time by Mason who introduced the very fertile concept of "unilateral gain." Further refinements have been supplied by Schaug-Pettersen and Towning, Garg<sup>4</sup> Dasher, and Youla<sup>5</sup>.

D. Youla and two of his colleagues<sup>6</sup> gave a very thorough and systematic discussion of the algebraic-geometric aspects of the theory and introduced the notion of a "cross-ratio" matrix of four  $n$ -ports. It was proved that this  $n \times n$  cross-ratio matrix undergoes a similarity transformation whenever the four  $n$ -ports are subjected to the same reciprocal  $2n$ -port embedding. With the aid of this result it is possible to attach to any nonreciprocal, non-lossless  $n$ -port  $N$  a characteristic  $n \times n$  matrix  $\Omega$  which also undergoes a similarity transformation when  $N$  is subjected to lossless, reciprocal  $2n$ -port embedding. Thus the  $n$  eigenvalues of this matrix emerge as physical invariants of the structure. Their true meanings can only be revealed by the discovery of a canonic form for  $N$ . Although some special canonic forms were obtained in ref. (4) and (6) the problem remained open. Recently, Tor-Schaug-Pettersen succeeded in deriving a canonic form for  $n$ -ports  $N$  which are either purely passive or purely active, however, it appears that at least part of his argument must be reviewed.

We are now in a position to summarize the main contributions of this paper. First, a new description of an  $n$ -port is introduced, the  $Q$  matrix, which always exists, provided the network is linear and time-invariant. The conditions of reciprocity, losslessness, etc., are very easily translated onto  $Q$  with the help of the three Lorentz spin matrices. If  $N$  is embedded in a  $2n$ -port  $M$  with chain matrix  $T$ , the  $Q$  matrix,  $W$ , of the resultant  $n$ -port,  $\hat{N}$  is given by  $W = TQ$ , i. e., by simply multiplying  $Q$  on the left with the  $2n \times 2n$  matrix  $T$ . A canonic form

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4. J. M. Garg, "Network Theory of Semiconductor Hall Plate Circuits" PIBMRI-809-60.
  5. D. C. Youla, "Some Results in the "Spot" Frequency Theory of Linear Noiseless, Time-Invariant  $N$ -ports." NEREM, 1960.
  6. D. C. Youla, L. Kaplan and D. Stock, "Analytical Time-Variant  $N$ -ports". PIBMRI-1028-62.



under lossless, reciprocal  $2n$ -port embedding is obtained for a large class of  $n$ -ports  $N$ . This class includes many active networks. A representation is established for a linear, noisy, time-invariant  $n$ -port and it is shown, that in general, one must include, externally, in each port, both series noise voltage generators and shunt current generators. The question of coherence and incoherence is also examined and compared with the recent results of Dahlke and Penfield. Lastly, the Haus-Adler theory of noisy  $n$ -ports is placed in a more general setting and canonic forms are obtained under  $2n$ -ports embeddings that are both lossless and reciprocal instead of merely lossless. The changes entailed by the additional restriction of reciprocity are analyzed at some length.

### III. PUBLICATIONS

The following publications by members of the Polytechnic Faculty reflect their activity in the areas of research supported by AFOSR Contract AF-18(603)-105.

#### A. Active and Time-Varying Systems

"Adaptive Control Systems" Edited by E. Mishkin and L. Braun, Jr. (New York: McGraw-Hill Book Co., 1961). This book written by research participants at the Polytechnic on their own time, has been utilized as a desirable and effective medium for the unified publication of research results which constitute techniques for analysis and design of active and feedback systems.

Additional publications of recent date include the following books:

- (1) J. G. Truxal, "Control System Theory" (New York: McGraw-Hill Book Company, 1960).
- (2) J. G. Truxal and W. A. Lynch, "Feedback Theory" (New York: McGraw-Hill Book Company, 1961).
- (3) J. G. Truxal and W. A. Lynch, "Introductory System Analysis" (New York: McGraw-Hill Book Company, 1961).
- (4) J. G. Truxal and E. Mishkin, "Identification Techniques" (New York: John Wiley and Sons, Inc., 1961).
- (5) J. G. Truxal and W. A. Lynch, "Signals and Systems in Electrical Engineering" (New York: McGraw-Hill Book Co., 1961).
- (6) J. G. Truxal, M. Shooman and W. B. Blesser, Chapter I in "Handbook of Telemetry and Remote Control," Editor Gruenberg, McGraw-Hill Book Co., 1961.

Journal articles and contributions to Proceedings from work directly under this contract include:

- (1) E. Mishkin and J.G. Truxal, "On the Evaluation of an Attitude Control System. Proc. of the Symposium on Active Networks and Feedback Systems, Polytechnic Institute of Brooklyn, April 1960.
- (2) J. G. Truxal, "Computers in Automatic Control Systems" (Proc. IRE, Vol. 49, No. 1 pp 305-312, Special issue on Computers, January, 1961).
- (3) E. Mishkin and J.G. Truxal, "On Identification and Evaluation of the transfer Dynamics of Physical Systems" in "Systems: Research and Design" Proceedings of Case Institute of Technology Symposium on Systems Engineering, Editor D.P. Eckman, John Wiley and Sons, Inc. 1961.
- (4) J.G. Truxal, L. Braun, Jr. and E. Mishkin, "On the Approximate Identification of Process Dynamics in Computer-Controlled Adaptive Systems" (Butterworth Scientific Publications, Ltd.; London, England, January 1961)
- (5) J.G. Truxal, "Control Engineering, Past and Present" Proc. of IRE Anniversary Issue, July 1962.
- (6) H. Ur, "Root Locus Properties and Sensitivity Relations in Control Systems." IRE Trans. Automatic Control, Vol. AC-5, No., Jan. 1960.

B. Linear Networks

- (1) Bounded Real Scattering Matrices and the Foundations of Linear Passive Network Theory, D. Youla, L. Castriota and H.J. Carlin, IRE Trans. PGCT, Vol. CT-6, No. 1, March 1959.
- (2) A Stability Characterization of the Reciprocal Linear Passive N-port, D. Youla, Proc. IRE Vol. 47, No. 6, June 1959.
- (3) Physical Realizability Criteria, D. Youla, IRE Convention Record, Part 2, 1960, pp 181.
- \* (4) W. Rubin and H.J. Carlin, Cascade Synthesis of Non-reciprocal Lossless Two-Ports, IRE Trans. PGCT.

\* Please note that Item 4 did not include a formal acknowledgement of Air Force Office of Scientific Research and Contract No. AF18(603)-105 support. It is regretted that this was omitted through a clerical error.

IV. TECHNICAL REPORTS AND MEMORANDA

The following reports have resulted from OSR sponsored research under contract No. AF-18(603)-105.

REPORT NO.	TITLE	AUTHOR
573-57	Multisection Microwave Terminating Structures 4-24-57	L. Coltun
594-57	Scattering Matrices and the Foundations of Linear, Passive Network Theory 9-10-57	D. C. Youla L. Castriota H.J. Carlin
697-58	Differential Properties and Sensitivity of Root Loci, Systems and Control Memo. No. 4 11-21-58	H. Ur
709-58	A Stability Characterization of the General, Linear, Passive, Reciprocal Time-Invariant N-Port Networks and Waveguides Memo. No. 17 1-9-59	D. Youla
807-60	Some New and Useful Matrix Results Networks and Waveguides Memo. No. 34 3-7-60	D. Youla
809-60	Network Properties of Three-Terminal Hall Plates, Networks and Waveguides Memo. 35 3-9-60	J. M. Garg
866-60	The Significance of Sensitivity in Feedback Systems Studies September 1961	W. A. Lynch J. G. Truxal
868-60	Network Theory of Semiconductor Hall Plate Circuits - June 1961	J. M. Garg
879-60	Computers in Automatic Control Systems 11-9-60	J. Truxal
908-61	Short-Time Stability in Linear Time-Varying Systems - 5-9-61	P. Dorato
920-61	Stability Analysis of Two-Parameter Control Systems - 6-6-61	M. Shooman
944-61	Stabilization of Multiloop System via the Sensitivity Function - 8-30-61	R. Haddad
945-61	On the Optimum Design of Predictor Control Systems - 8-24-61	S. Horing

REPORT NO.	TITLE	AUTHOR
1004-62	Pontryagin's Maximum Principle: Boundry Conditions From a Dynamic Programming Formulation	P. Dorato
1005-62	Plant-Adaptive Optimal Systems	P. Dorato
1015-62	Application of Pontryagin's Maximum Principle: Linear Control Systems	G. L. Collina P. Dorato
1016-62	The Analysis and Synthesis of RC Synchronous Networks	B. M. Tobin
1068-62	On the Synthesis of Resistor N-Ports	F. T. Boesch
1075-62	On the Design of Linear Process Adaptive Control Systems	H. N. Yagoda

A set of abstracts for each of the above is appended.

#### V. FACULTY

This program has been under the direction of Professor John G. Truxal. Professor Herbert J. Carlin, Head of the Department of Electrophysics has cooperated with Professor Truxal particularly on the phase concerned with linear network studies. Other members of the faculty who have contributed to this research program are:

Francis T. Boesch, Instructor, Electrical Engineering  
 Ludwig Braun, Jr., Assoc. Professor, Electrical Engineering  
 Liborio J. Castriota, Lecturer in Electrical Engineering  
 Peter Dorato, Assistant Professor, Electrical Engineering  
 Richard Haddad, Assistant Professor, Electrical Engineering  
 Sheldon Horing, Assistant Professor, Electrical Engineering  
 William A. Lynch, Professor, Electrical Engineering  
 Eliezer Mishkin, Professor, Electrical Engineering  
 Martin L. Shooman, Assistant Professor, Electrical Engineering  
 Dante C. Youla, Research Associate Professor, Electrophysics

POLYTECHNIC INSTITUTE OF BROOKLYN

573

MULTISECTION MICROWAVE TERMINATING STRUCTURES,  
by L. Coltun

ABSTRACT

Analysis and methods of design are presented for several special broadband transmission line terminating structures. Each is characterized by a tandem connection of subdivisions called "unit cells" - where each cell consists, in general, of a lossy shunt admittance and several lossless transmission lines having non-ferrous dielectrics.

An approach toward broadband structures of shorter lengths is described in which the input admittance to each cell is real at the center frequency of design.

Report No. 573  
AD 132 357

4/ 24/ 57  
51 Pages

Contract No. AF-18(603)-105  
Net

Electrophysics

SCATTERING MATRICES AND THE FOUNDATIONS OF LINEAR,  
PASSIVE NETWORK THEORY, by D.C. Youla, L.J. Castriota and  
H.J. Carlin

ABSTRACT

This report is concerned with the construction of a rigorous theory of linear passive networks from the point of view of energy conservation and causality. It is shown that five postulates defining the physical nature of a network suffice to give all analytic properties of the network scattering matrix  $S(z)$ : ( $z$  is the complex frequency variable  $\omega + i\beta$ ). These analytic properties define a bounded real scattering matrix, the possession of which is necessary and sufficient for an  $n$ -port network to satisfy physical realizability as stated by these postulates. A bounded real scattering matrix is analytic and bounded in the strict upper half of the  $z$  plane, and has an associated form  $Q(z) = I_n - \overline{S}^T(z) S(z)$  which is the matrix of a positive hermitian form in this region. An alternate formulation of the bounded real property is given which stresses boundary behavior of  $S(z)$  and  $Q(z)$ . These analytic properties of  $S(z)$  lead to a generalized formulation of a positive real immittance matrix.

A theorem concerning the necessary and sufficient conditions for the existence of a Faltung-representation is proved and its implications discussed. The entire treatment is mathematical in nature and leans heavily on the theory of linear bounded operators.

Report No. 594  
AD 136 569

9/ 10/ 57  
77 Pages

Contract No. AF-18(603)-105  
Net

Electrophysics

POLYTECHNIC INSTITUTE OF BROOKLYN

697

DIFFERENTIAL PROPERTIES AND SENSITIVITY OF ROOT LOCI,  
by Hanoch Ur

ABSTRACT

The differential properties of root loci, including pole sensitivity, angle of slope and curvature of the locus, are investigated in a unified manner. The locus is treated as a transformation of a line (the real axis) in the  $K$  plane to the  $s$  plane and properties of analytic functions are used to simplify calculations and results. It is shown that all results apply to the general root locus of a non real  $K$ .

Report No. 697  
PIB No. 625

11/21/58  
24 Pages

Contract No. AF-18(603)-105  
Net

Systems and Control

S and C Memo No. 4



POLYTECHNIC INSTITUTE OF BROOKLYN

709

A STABILITY CHARACTERIZATION OF THE GENERAL LINEAR,  
PASSIVE, RECIPROCAL, TIME-INVARIANT n-PORT, by  
D.C. Youla

ABSTRACT

The fact that a two port which is reciprocal and stable under all passive terminations is passive has been known for quite some time. A typical proof of this result is given in a recent paper by E. Folke Bolinder entitled, "Survey of Some Properties of Linear Networks", Sept. 1957, Transactions of the P.G.C.T. In this memorandum it is shown that the theorem remains valid for arbitrary n-ports, i.e., a reciprocal n-port stable under all passive terminations is passive. The technique employed makes use of induction and the properties of mappings which transform the right half-plane into the right half-plane. In addition, the scattering matrix  $S(p)$  of the structure is assumed to be analytic and of "limited growth" (defined later) in  $\text{Re } p > 0$ , no assumptions being made regarding its boundary behavior on  $p = j\omega$ . Thus, non-lumped networks fall within the scope of the theorem.

Report No. 709  
PIB No. 637

1/9/59  
15 Pages

Contract No. AF-18(603)-105  
Net

Networks and Waveguides

N and W Memo No. 17

## SOME NEW AND USEFUL MATRIX RESULTS, by D.C. Youla

ABSTRACT

Let  $C$  be a square matrix with complex elements: If  $C = C'$  ( $C'$  denotes the transpose of  $C$ ) there exists a unitary matrix  $U$  such that

$$U'CU = \text{diag} (\mu_1, \mu_2, \dots, \mu_n)$$

where the  $\mu$ 's are the non-negative square roots of the eigenvalues  $\mu_1^2, \mu_2^2, \dots, \mu_n^2$  of  $C^*C$  ( $C^*$  is the adjoint of  $C$ ) (2). If  $C$  is skew-symmetric, that is,  $C = -C'$ ,

$$U'CU = \sum_1 \dot{+} \dots \dot{+} \sum_k \dot{+} 0 \dot{+} \dots \dot{+} 0$$

where

$$\sum_r = \begin{bmatrix} 0 & a_r \\ -a_r & 0 \end{bmatrix}, \quad a_r > 0, \quad r = 1, 2, \dots, k,$$

and the  $a$ 's are the positive square roots of the non-zero eigenvalues  $a_1^2, a_2^2, \dots, a_k^2$  of  $C^*C$  (1). Clearly  $\text{rank } C = 2k$  and the number of zeros appearing in (2) is  $n - 2k$ . Both (1) and (2) are classical. In a recent paper (3) Stander and Wiegman, apparently unaware of (1), give an alternative derivation of (2) with its appropriate generalization to quaternions.

The forms appearing on the right-hand sides of (1) and (2) possess the distinction of actually being canonic for symmetric and skew-symmetric matrices, respectively. The problem of finding a canonic form for an arbitrary matrix under the group of unitary congruence transformations is not only of mathematical interest but of the utmost importance for applied electrical engineering network theory, representing as it does the physical operation of embedding an  $n$ -port in a lossless, reciprocal "all-pass"  $2n$ -port (4;5). In this paper a start is made on this problem by obtaining a normal form for an arbitrary matrix under a  $U'$ ,  $U$  transformation and it is shown that (1) and (2) are immediate corollaries.

Report No. 807

3/7/60  
21 Pages

Contract No. AF-18(603)-105  
Net

Networks and Waveguides

N and W Memo No. 34

NETWORK PROPERTIES OF THREE-TERMINAL HALL PLATES,  
by J.M. GargABSTRACT

Based on the no-voltage-amplification property of devices in which Hall-effect is present, an idealization (which can be closely approached in practice) is postulated. The necessary and sufficient conditions on a constant real non-symmetric  $2 \times 2$  matrix that it represent this 3-terminal gyrator-like element are derived. An equivalent circuit is proposed which is useful to represent the terminal performance of Hall-plates. Application of these results to derive minimum insertion-loss of a one-way-line using ideal three-terminal Hall-plates is also presented. It is shown that it is impossible to construct a 3-port circulator using only positive resistors and a 3-terminal Hall plate.

Report No. 809

3/9/60  
25 PagesContract No. AF-18(603)-105  
Net

Networks and Waveguides

N and W Memo No. 35

## THE SIGNIFICANCE OF SENSITIVITY IN FEEDBACK SYSTEM STUDIES, by W.A. Lynch and J.G. Truxal

ABSTRACT

The sensitivity function defined originally by Bode and Mason is the fundamental measure of the effectiveness of feedback, in controlling the effects of both parameter variations and unwanted, noise signals. After a preliminary discussion of the significance of the sensitivity function, methods for the simple calculation of the sensitivity are presented (Section 5) -- methods valid whenever the overall transmission is a bilinear function of the varying parameter (thus, whenever the parameter is a circuit element or the transmittance of a controlled source).

The sensitivity function yields simple evaluations of the parameter margins (Section 6) -- the amount by which specific parameters must be varied to cause system instability. The parameter margins can be evaluated directly from the frequency-domain plots utilized in normal system design. This interpretation of the sensitivity function is of importance not only in the logical design for simultaneous control of several sensitivity functions, but also in the identification problem. Extensions to the study of non-linear systems can be effected if a frequency-independent describing function can be defined.

Report No. 866  
AFOSR 1540

6/1/61  
111 Pages

Contract No. AF-18(603)-105  
Net

Electrical Engineering

NETWORK THEORY OF SEMICONDUCTOR HALL PLATE  
CIRCUITS, by J.M. GargABSTRACT

The Hall effect in semiconductors provides a purely electrical approximate physical realization of the ideal gyrator, particularly at low frequencies. This investigation is concerning the properties and synthesis of passive electrical networks which contain Hall plates (as these are called) as basic non-reciprocal elements. Network representation of some basic plates namely the general 3-terminal, and symmetric 4-terminal and 6-terminal plates is given.

The report also deals with the construction of some useful nonreciprocal devices such as isolators, gyrators, and circulators after suitably modifying the properties of Hall plates by conventional circuit elements. The transforming (or embedding) networks have been considered in two broad classes: The lossy transformerless, frequency invariant (or pure resistive) type; and the lossless reciprocal type. The resistive embedding has been known and used earlier, but this appears to be the first systematic attempt to optimize the embedding networks and consider the effects of mismatch upon the insertion loss of devices. An improvement in the theoretical minimum forward insertion loss of a matched 3-port circulator from an earlier figure of 8.4 db to about 7 db has been obtained.

Experimental results are given on actual operation of the more promising devices such as an isolator constructed from a symmetric 4-terminal plates by lossy and LR embedding (7.5 db and 4.6 db respectively); and circulators of forward loss 8.1 db and 5.8 db (constructed from a symmetric 6-terminal plate by lossy lossless-reciprocal embedding respectively) and actual operation of a low-frequency nonreciprocal tunnel diode amplifier.

This study establishes the best that could be expected from Hall plates, and that in order to be more attractive the use of negative resistors may be required. In conclusion some unsolved problems concerning effect of magneto-resistance, higher order circulators from a single plate etc., are pointed out for future work.

Report No. 868  
AFOSR 693

6/61  
118 Pages

Contract No. AF-18(603)-105  
Net

Electrophysics

POLYTECHNIC INSTITUTE OF BROOKLYN

879

COMPUTERS IN AUTOMATIC CONTROL SYSTEMS, by  
J.G. Truxal

ABSTRACT

Modern automatic feedback control systems increasingly utilize flexible and highly developed electronic computers as active controllers. The trend from the use of simple electric circuits and elementary analog devices toward high-speed digital computers (or computers employing both digital and analog devices) is accelerated by: (1) the rapidly growing complexity of modern control systems; (2) the development of optimizing control; and (3) the increasing importance of self-adaptive control. Recent applications of computer control include the automatic control of boiler operation in an electric generating station, the adaptive autopilots for piloted aircraft, and the automatic control of load dispatching in electric power distribution.

Report No. 879  
AFOSR-TN-60-1475

11/ 9/ 60  
44 Pages

Contract No. AF-18(603)-105  
Net

Systems and Control

SHORT-TIME STABILITY IN LINEAR TIME-VARYING SYSTEMS,  
by P. Dorato

ABSTRACT

The classical stability concepts of Lyapunov and Poincaré deal with systems operating over an infinite time interval. In contrast, short-time stability deals with systems operating over a finite time interval. In this dissertation short-time stability is investigated in linear time-varying systems. This concept of stability finds application for example, in missile and satellite systems where operating times are often of finite duration. Short-time stability assures, in a finite time interval  $0 < t < T$ , that all prescribed constant  $C$ . The values of  $\epsilon$ ,  $C$ , and  $T$  are determined by the particular application at hand.

The study of short-time stability is divided into two categories: undriven systems and driven systems. Undriven systems are represented by the set of differential equations:

$$\dot{y}_s = a_{s1}(t) y_1 + a_{s2}(t) y_2 + \dots + a_{sn}(t) y_n, \quad s = 1, 2, \dots, n.$$

Sufficient conditions for short-time stability are then given in terms of the coefficients,  $a_{ij}(t)$ .

Driven systems are represented by their impulse response  $w(t, \tau)$ . A necessary and sufficient condition for short-time stability in driven systems is given directly in terms of  $w(t, \tau)$ . Sufficient conditions for short-time stability in feedback systems, in terms of the open loop impulse response  $g(t, \tau)$ , are also included. In addition the concept of short-time C-equivalence, essentially a structural stability concept, is introduced. Necessary and sufficient conditions for two systems to be short-time C-equivalent are presented.

The results are applied to various examples of linear time-varying and time-invariant systems.

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Electrical Engineering

STABILITY ANALYSIS OF TWO-PARAMETER CONTROL SYSTEMS,  
by M. L. ShoomanABSTRACT

This dissertation treats the problem of stability in two-parameter control systems. The two parameters may be loop-gains, time constants, natural frequencies, damping constants, or describing function gains representing system nonlinearities. The limitations of single parameter analysis techniques are discussed and parameter plane analysis introduced.

The two system parameters are plotted along the coordinate axes in the parameter plane. The left-half-plane and the  $j\omega$  axis of the  $s$ -plane are mapped into a stable region and a stability boundary respectively, in the parameter plane. The parameter plane plot delineates the ranges of the two system parameters that yield stable operation. The Routh Table or other similar stability criteria are used to obtain the stability boundary in the parameter plane. Using appropriate mapping transforms, performance boundaries may be obtained. These transformations allow a mapping of lines of constant  $\sigma$  in the left-half-plane into the parameter plane. Analog computer methods are discussed that allow experimental location of stability and performance boundaries in complex problems.

In Chapter 4, the parameter plane method of analysis is applied to a system with two nonlinearities. A limit cycle oscillation is predicted for the system using parameter plane analysis. The system is simulated on an analog computer, the oscillatory behavior displayed, and the frequency and amplitudes of oscillation predicted. A theoretical and computer study of system compensation is made. The final system with the chosen compensation is found to yield well behaved responses to a wide range of excitation amplitudes.

The concluding chapter discusses three means of analyzing three and more parameter systems. While none of the three methods yield an analysis that is in general satisfactory, as a group they provide a valuable approach to a wide class of multiple parameter problems.

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Electrical Engineering



STABILIZATION OF MULTILOOP SYSTEMS VIA THE  
SENSITIVITY FUNCTION, by R. Haddad

ABSTRACT

The problem studied in this thesis is the stabilization of multiloop systems against individual variations in one or more of the system parameters. A correlation between the stability of a multiloop configuration and the real-frequency behavior of the sensitivity function is derived. From this relationship, a set of stability margins is developed which provide a quantitative measure of the destabilizing effects of variations in each parameter. The margins, referred to as "parameter gain and phase margins" are shown to be generalizations of the familiar gain and phase margins of the single-loop, servo system.

The design objective is the realization of a set of specified stability margins; the relationship between these margins and the real-frequency behavior of the associated sensitivity functions gives rise to specification of the sinusoidal response of compensation networks at a set of discrete frequencies.

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Electrical Engineering

ON THE OPTIMUM DESIGN OF PREDICTOR CONTROL SYSTEMS,  
by Sheldon Horing

ABSTRACT

The purpose of this dissertation is to investigate the optimum design of Predictor Control Systems subject to statistical or deterministic inputs, under appropriate constraints on the control signal. The control signal constraints treated in this dissertation are:

1. The magnitude of the control signal equal to a constant. This is the Bang-Bang control problem.
2. The magnitude of the control signal less than or equal to some constant.

The basic optimization problem is viewed, in very general terms, as a problem in decision theory. Based on available information, the decision must be made as to which of the allowable control signals should be applied to the plant in order to achieve the optimum performance. A sum of squared error criterion is used to define the optimum in order to reduce the problem to a problem in linear decision theory. A piecewise-linear Control Boundary is found which can be used to generate the desired control signal. The realization of this Control Boundary is shown to be straightforward and inexpensive. This technique can be applied to fixed linear plants of arbitrary order. A linear approximation to the optimum Control Boundary is discussed. The practical advantages of such an approximation are shown to be considerable.

Experimental results verifying the techniques which are developed are presented. The results of an experimental investigation of the sensitivity of the actual mean-squared error to plant parameter variations are included among these.

In all, this dissertation presents a somewhat different approach to the optimization problem.

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POLYTECHNIC INSTITUTE OF BROOKLYN

1004

PONTRYAGIN'S MAXIMUM PRINCIPLE: BOUNDARY CONDITIONS  
FROM A DYNAMIC PROGRAMMING FORMULATION, by P. Dorato

ABSTRACT

The boundary conditions required in Pontryagin's maximum principle are derived from a dynamic programming formalism.

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Systems and Control

S and C Memo No. 40

POLYTECHNIC INSTITUTE OF BROOKLYN

1005

PLANT-ADAPTIVE OPTIMAL SYSTEMS, by P. Dorato

ABSTRACT

It is demonstrated that Pontryagin's maximum principle yields an adaptive control law, once the plant parameters are introduced as identifiable elements.

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Systems and Control

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1015

APPLICATION OF PONTRYAGIN'S MAXIMUM PRINCIPLE:  
LINEAR CONTROL SYSTEMS, by G.L. Collina, and P. Dorato

ABSTRACT

This study is concerned with the problem of linear, continuous-time, deterministic optimal, control systems. In particular it is shown, via Pontryagin's maximum principle, that linear optimal control systems always result from the combination of (1) a linear plant; (2) integral quadratic performance criterion; and (3) no constraints on the plant input. The controller synthesis is outlined in the general time-varying case and various illustrative examples are included. A comparison of the maximum principle approach with dynamic programming and the classical calculus of variations is also included. Both driven and undriven control systems are considered. For the case where the time-interval of optimality is infinite, the stability of the optimal system is investigated. It is shown that stability is assured whenever the integral of the performance criterion is positive definite.

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Electrical Engineering

THE ANALYSIS AND SYNTHESIS OF RC SYNCHRONOUS NETWORKS,  
by B.M. Tobin

ABSTRACT

A major problem in the field of a-c control systems is the synthesis of suitable electrical compensating networks which operate on the envelopes of modulated signals. Classical time-invariant compensating networks suffer from a number of disadvantages.

1. In general the networks are realizable only as RLC structures which often require unreasonably high-Q coils.
2. Practical RC Networks can realize only lead compensation.
3. The network response is sensitive to variations in carrier frequency.

RC synchronous networks in which the basic reactive elements are synchronously-switched capacitances whose current polarities reverse periodically at twice the carrier frequency overcome these drawbacks. Although successful single capacitance synchronous networks have been constructed an exact theory upon which a logical synthesis procedure can be based has been lacking.

This report presents a unified theory of analysis and synthesis of RC synchronous networks. The theory leading to a rigorous procedure for synthesizing an RC synchronous network which realizes a prescribed envelope voltage transfer function is developed in the following sequence of steps.

1. An idealized linear model of the synchronously-switched capacitance is postulated; the non-instantaneous reversal of capacitance current and the possibility of unequal intervals of time during which the capacitance current remains interrupted are accounted for. On the basis of this model an exact expression is derived for the transient and steady-state response of a synchronous network containing a single synchronously-switched capacitance imbedded in an otherwise arbitrary resistive network. The analytical technique utilizes constant coefficient, first order, linear difference equations which are solved with the aid of the Z-transform.

THE ANALYSIS AND SYNTHESIS OF RC SYNCHRONOUS NETWORKS,  
by B.M. Tobin

ABSTRACT (continued)

2. The envelope driving-point immittance of a synchronously-switched capacitance imbedded in a resistive network is obtained and is shown to be substantially independent of carrier frequency for narrow envelope bandwidths. Unfortunately the immittance proves to be a function of the Thévenin resistance facing the synchronously-switched capacitance. The situation is aggravated to unreasonable proportions in multi-capacitance networks. Any hope for treating the synchronously-switched capacitance as a basic network element is negated.
3. An expression is derived relating the envelope voltage transfer function  $G_s(s)$  of an arbitrary RC synchronous network to the signal voltage transfer function  $G_o(s)$  of its time-invariant counterpart. Although a narrow envelope bandwidth with respect to the carrier frequency is assumed, it is shown that this leads to relatively inconsequential errors in practical systems.
4. From the relationship between  $G_s(s)$  and  $G_o(s)$  a logical procedure is developed for the exact synthesis of RC synchronous networks which realize a prescribed envelope voltage transfer function.

The report concludes with the presentation of a general method for the analysis of any RC network containing a plurality of periodically switched, piece-wise constant elements.

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Electrical Engineering

POLYTECHNIC INSTITUTE OF BROOKLYN

1068

ON THE SYNTHESIS OF RESISTOR N-PORTS, by F.T. Boesch

ABSTRACT

The problem of determining the necessary and sufficient conditions for a matrix  $Y$  to be the short-circuit admittance matrix of a transformerless resistor  $n$ -port has been treated extensively in the literature. The major effort in the treatment of the problem has been directed toward the case of an  $n + 1$  terminal structure. This memo will summarize the contributions to this case, and a new procedure will be presented which has the advantage of being considerably simpler than any previous solution.

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ON THE DESIGN OF LINEAR PROCESS ADAPTIVE CONTROL SYSTEMS,  
by H. N. Yagoda

ABSTRACT

This dissertation treats the problem of controlling a stable, slowly time varying, linear process. A solution is proposed that is simple, practical and applicable to any finite order process. Use is made of a continuously adjusted tandem compensator; a comparison of the weighted histories of the input and output signals is used to control this adjustment. System design is based upon the use of a variable transfer function characterization for both the compensator and the process. In effect, a compensator is designed that attempts to cancel the variable poles of the process with variable zeros, the variable zeros of the process with variable poles and the variable process gain with its reciprocal.

The design technique developed is applied to several problems. The processes involved range from a variable gain amplifier through a plant that contains a variable gain and two pair of variable complex poles. The processes are grouped in accordance with the number of variable parameters in the process characterizing function.

Several adaptive processes are simulated on a computer and the resulting operation of the adaptive circuitry is presented for comparison with the theoretically predicted operation. Among the processes investigated are:

1. a variable gain amplifier
2. a process with two variable real poles
3. a process with a variable gain and two pair of variable complex poles.

The adaptive circuitry is modified for the process with two variable real poles and the resulting operation of that adaptive process is presented. The agreement between the theoretically predicted results and the operation of the simulated systems is good.

The amenability to analysis of the technique presented, allows for the investigation of the effect of noise on the adaptive process. In addition, stability is investigated. Neither of these appears to offer a major problem in the operation of the proposed control systems.

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